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Analysis of Tunnels Using Latin Hypercube Sampling

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ABSTRACT: The paper is focused on the Latin Hypercube Sampling method (LHS) which can be successfully utilised for description of the variability of the soil and rock mass in the geotechnical design. It gives a brief description of the input parameters as random quantities and describes the LHS-mean method in detail with the focus on methods for statistical dependence. The main part of the paper is then focused on the performed application of LHS in the analysis on several tunnels constructed in the Czech Republic.

1 Introduction

Parameters of soils and rocks obtained for the geotechnical structures design within the framework of geotechnical investigation display significant scatter. Uncertainties in input parameters in the deterministic calculations are considered by means of safety factors. In this standard conception the results are safe and conservative, but may substantially differ from the real behaviour. The probabilistic methods take into consideration the input parameters variability and offer determination of the possibility to determine the probability of s defect and the index of reliability. Therefore the probability methods have become the tool for solving geotechnical problems in present engineering practice. The following paper deals with the Latin Hypercube Sampling method, which is a good alternative to the time-demanding Monte Carlo simulation method. The paper presents a short description of the LHS method and its application in the Czech tunnelling practice.

2 Random variables

Results of numerical modelling of geotechnical problems are very sensitive to input parameters (in the content of the probabilistic analysis we can speak about random variables). The uncertainty associated with the determination of parameters leads to the uncertainty in the results of the solved problem. For their formulation it is advisable to take into consideration their character of random quantities. A random quantity assumes various values and is described by the probability density distribution. From the practical point of view, it is good to take into consideration only continuous random quantities, i.e. those which can assume all values from particular interval. Firstly we have to select a statistical distribution of the input parameters in the engineering practice. Many authors have shown that the rock mass parameters can be well represented by a normal distribution (Sari, 2009). On the other hand, we know from practice that this is not general rule (for example fiction angle, joint length and waviness angle have a log-normal distribution). If a data from laboratory or in-situ tests are known we can easily find the best fitting distribution by application of statistical software. For the statistical processing of the measured properties of rock or soil specimens we used QC-Expert software with probability module (Trilobyte, 2012). This module provides the MLE method (Maximum Likelihood Estimate) for deriving best estimations for the given data. In general, MLE method selects parameters that produce a distribution that fits the observed data the best (i.e., parameters for a given statistic that make the likelihood function a maximum).

3 Latin Hypercube Sampling

To reduce the number of samples required for sufficient accuracy in Monte Carlo simulation other sampling methods have been developed. One of the best is the Latin Hypercube Sampling method. The concept of LHS is based on the Monte Carlo techniques. Latin Hypercube Sampling preserves marginal probability distributions for each simulated variables. To fulfil this aim, Latin Hypercube Sampling constructs a highly dependent joint probability density function for the random variables in the problem. This allows adequate accuracy in the response parameter using only a small number of samples. LHS is a form of stratified sampling that can be applied to multiple variables. The method is commonly used to reduce the number of runs necessary for a Monte Carlo simulation to achieve a reasonably accurate random distribution. A Latin square is a square grid containing sample positions when there is only one sample in each row and each column. A Latin hypercube is the generalisation of this concept - each sample is the only one in each axis. . There are more methods available for the samples selection so we distinguish several kinds of LHS: random, mean, median, etc. Figure 1 depicts the cumulative distribution function F(x) which is divided into eight intervals with equal probability of 1/N. Points on vertical axis represent midpoints of these intervals and subsequently mark the corresponding values of the variable X.

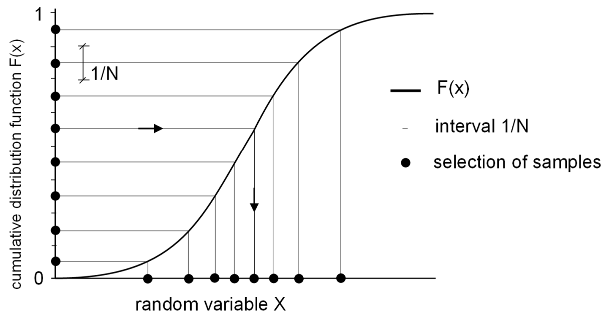

Figure 1. The principle of the LHS-median method sampling

The maximum number of combinations in LHS can be generally expressed by the following equation:

$$\prod_{n=0}^{N}(M-n)^{N-1}a \tag{1}$$

where:
M= number of divisions (sample points)
N= number of variables

3.1 *Method for statistical dependence*

It is advisable to verify the measure of statistical dependence of input parameters (columns of the permutation table) using statistical correlation. There are several methods for selecting samples proposed for the simulation of correlated random vectors available within the LHS simulations. The majority of works solve the implementation of correlation in the form of exchanging the sequence of samples at individual variables and do not change their values. In this way, the probability distribution of each random quantity is preserved.

The widely spread method for introducing correlation in the LHS method is the orthogonal transformation of independent random variables into correlated variables. The basis of this principle was first published by Scheuer and Stoller (1962). They proposed a method for generating correlated variables following the Gaussian curve of distribution, which is based on Cholesky decomposition of covariance matrices.

Iman and Conover (1982) proposed a more general method using Spearman's coefficient of correlation. This method is not so restricted to normally distributed random variables. Nevertheless, the assumption of uncorrelated input variables and a persisting significant error in correlation between variables with respect to the required correlation matrix remains to be a disadvantage.

The last published method (Florian 1992) starts from the Iman and Conover procedure and was developed for the reduction of undesired correlation with the aim of obtaining uncorrelated variables.

4 LHS application in the Czech Republic

The majority of initial works dealing with use of the LHS method was carried out on university premises in the Czech Republic. The statistical analysis of the tunnels in Prague will be described in the following sections, but we have to mention the other significant applications of the LHS. In the tunnel Valík calculation (Hrubešová et al. 2003) LHS was applied to the assessment of the influence of 10 parameters (e.g. the anisotropy coefficient, dipping of the discontinuities etc.). Vaněčková (2008) used the LHS method for solving problems of stability of rock slope interspersed by a system of discontinuities. Also Parák (2008) used the LHS method for the determination of the influence of geotechnical parameters of geological strata, parameters of sprayed concrete and initial conditions on structural forces in a circular tunnel lining and Pruška and Šedivý (2010) described LHS application in the slope stability problems.

4.1 *Modelling of the Mrázovka tunnel*

The outputs of the Mrázovka tunnel modelling were verified by the Latin Hypercube Sampling method (Barták 2003). The numerical analysis was carried out by means of the PLAXIS program. The 3D behaviour of the excavation face area, and correct description of the influence on the deformations and the condition of the massif were simulated by the common procedure of loading the excavation and lining using the so-called β-method (Fig. 2). The modelled area of the profile was approximately 200 m wide and 110 m high and was divided into eight basic sub-areas according to the types of rock encountered (Fig. 3). The rock mass behaviour was approximated by means of the Mohr-Coulomb model.

The input geotechnical parameters of the rock mass (E_{def}, ν, c, φ, γ) were determined on the basis of the engineering-geological investigation, see Table1.

A comparison of theoretically determined deformations with the values obtained by monitoring was used to verify the applicability of the mathematical model. The results of the statistical study of the West Tunnel Tube (WTT) in profile km 5.160 are shown in Table 2. They show that the probability of final settlements being between 71 mm and 213 mm is 95%. The range of settlement without including deformations caused by the excavation of the pilot adit is 65 mm to 198 mm. The predicted range was consistent with the measured value of 194 mm (value does not include effect of pilot adit).

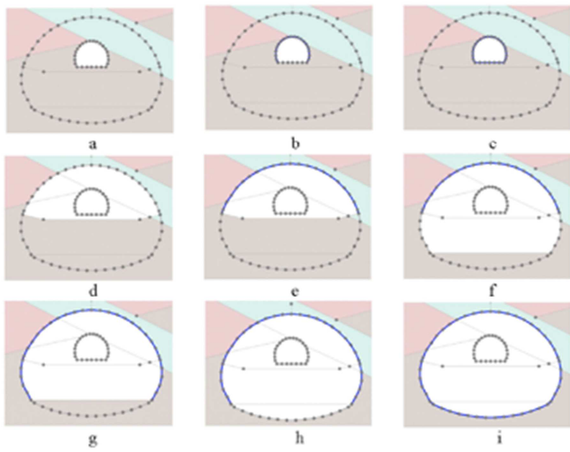

Figure 2. Numerical modelling stages

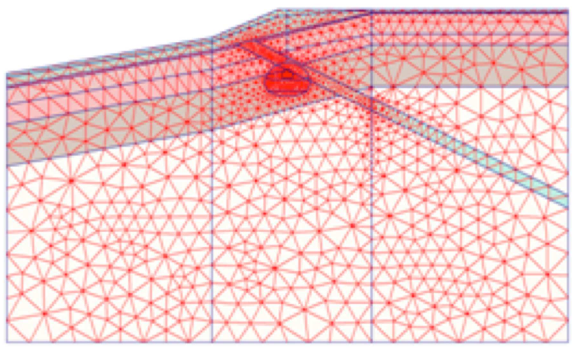

Figure 3. Geometry of generated mesh in WTT km 5.160

Table 1.Input parameters on the first run.

Parameter	E_{def} [MPa]	ν [-]	γ [kN/m^3]	c [kPa]	ϕ [^{0}]
Ground 1	4.8	0.30	17.3	11.3	26.0
Ground 2	13.6	0.30	18.8	5.3	28.0
Ground 3	24.2	0.35	20.2	14.1	23.5
Ground 4	79.7	0.31	22.6	22.3	27.5
Ground 5	171.0	0.29	24.3	45.5	32.5
Ground 6	271.0	0.26	25.1	91.0	36.5
Ground 7	421.0	0.20	25.8	91.0	36.5
Ground 8	24.5	0.33	23.1	13.2	26.5

Table 2.Results of the calculations (mm).

Calculation	Total Surface	settlement Tunnel	Adit Surface	settlement Tunnel
1	79	107	15	8
2	153	197	29	14
3	92	125	18	10
4	85	111	15	8
5	134	171	25	12
average	108.60	142.20	20.40	10.40

4.2 *Modelling of the Špejchar – Pelc-Tyrolka tunnel*

The Špejchar – Pelc-Tyrolka project is 4.320 m long with the length of the tunnelled section being 3.438 m (it will go into service in 2014). Apart from the tunnel, the project includes underground garages in Letná, four underground technology centres and the Trója Bridge. As designed, the excavation will be carried out using the New Austrian Tunnelling Method (NATM). Due to the predicted conditions, a vertical excavation sequence will be adopted in the three-lane tunnel excavation. In the two-lane tunnels, a horizontal excavation sequence is expected. The position of the exploratory drift has been selected to coincide with the complicated sections of the tunnel to provide information to predict whether the vertical sequence should be applied to the top heading only, or to the entire cross section. Mechanical rock breaking is expected in combination with the drill-and-blast when passing through the Quartzites. A transition zone between the Quartzites exists at the foot of the slope falling from the Letná Plain. The gradient parameters mean that the tunnels run in the vicinity of fully saturated Quaternary sediments. A multi-criteria analysis resulted in the selection of pre-excavation grouting.

The exploratory drift and tunnel tubes near the transition zone (see Figure 4) were verified by LHS method (Loužensky 2006, Kolařík 2008).The numerical models were carried out using the GEO 5 and CESAR - LCPC programs (see Figure 5). The rock mass behaviour was approximated by means of the Mohr-Coulomb model. The intervals of the input parameters of the rock mass were determined on the basis of the engineering-geological investigation results.

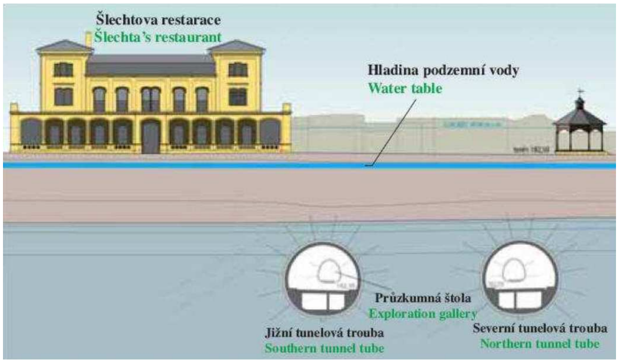

Figure 4. The Blanka tunnel cross section

The results of statistical study of South Tunnel Tube (STT) showed that the range of predicted surface settlement above excavated tunnel is from 2 mm to 23 mm. The results of the statistical study of the surface settlement at profile STT km 6,840 with in-situ measurement comparison are shown in Table 3.

Table 3.Results of the tunnel calculations - vertical def. (mm) in measured points on the surface.

Calc.	Point 123	Point 124	Point 239	Point 240	Point 037
1	-1.9	-15.2	-21.6	-13.4	-10.9
2	-1.6	-9.7	-14.3	-8.9	-8.6
3	-1.9	-14.3	-20.3	-12.7	-10.6
4	-1.6	-7.6	-13.4	-8.5	-8.2
5	-2.1	-8.7	-15.2	-9.9	-8.8
6	-1.8	-11.6	-20.1	-12.6	-10.2
*In-situ	-5.0	-24.0	-22.5	-8.0	-7.5

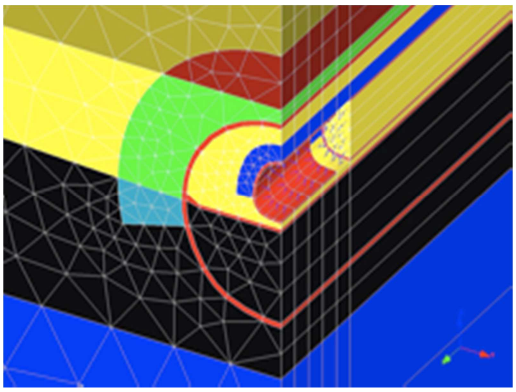

Figure 5. 3D model of the Špejchar – Pelc-Tyrolka tunnel (detail of the exploratory drift)

4.3 *Modelling of the Prague metro line A Extension*

Prague metro line A Extension is 6.134 m long and comprises of four stations. Initial works started in April 2010 and it will be opened in 2015. All underground stations (except Motol station) are mined using NATM. The running tunnel from Vypich to Motol (two tracks) is excavated using NATM, the other tunnels (single track) are driven by two tunnelling machines of the EPBS type. Above described LHS method was applied to verify a change in the Petřiny station construction concept. The Petřiny station is the single-nave mined station with cross section area of 266 m^2 and the length of 217 m – Figure 6. The station excavation sequencing consists of two side drifts and one central pillar due to very difficult geological conditions. The side drifts have cross section area of 70 m^2 and are subdivided into top heading, bench and invert.

The rock mass behaviour was approximated by means of the Mohr-Coulomb model. The input geotechnical parameters of the rock mass (E_{def}, ν, c, φ, γ) were determined on the basis of the engineering-geological investigation results, see Table 4. The coefficient of lateral pressure K_o was calculated according known equations - Janbu or using Poisson's number.

The numerical analysis was carried out by 3D FEM (Figure 7) using the MIDAS GTS program (Kožoušek 2012). The results of statistical study for Petřiny station are shown in Table 5 and graphically in Figures 8 and 9. They show that final settlements of the tunnel lining will be with probability 95% between values 10.9 mm and 13.5 mm. The interval of final surface settlement above excavated station is from 5.5 mm 6.9 mm.

Table 4. Input parameters for the modelling

Parameters	GT3 marlstones	GT6 sandstones	GT9 shales
E[MPa]	1800	750	900
ν[–]	0.4	0.25	0.25
γ[kN/m^3]	22	20	25
C [kPa]	150	100	100
φ [°]	40	38	40
K_0 (ac. ν)	0.25	0.33	0.33
K_0 (ac. φ)	0.36	0.38	0.36
E * 0.5	900	375	450
γ * 0.8	17.6	16	20
c * 0.5	75	50	50
φ * 0.5	20	19	20
ν * 0.8	0.3	0.2	0.2

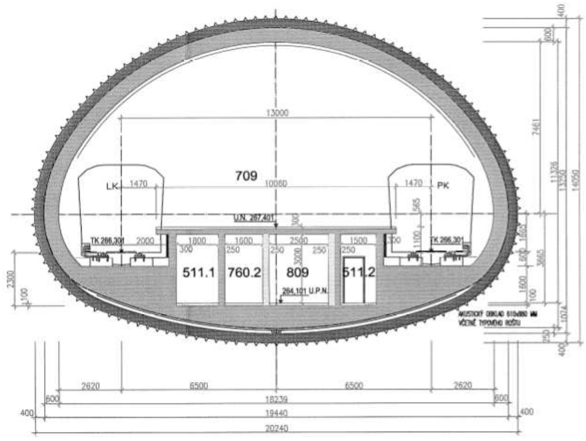

Figure 6. Cross-section of the Petřiny station

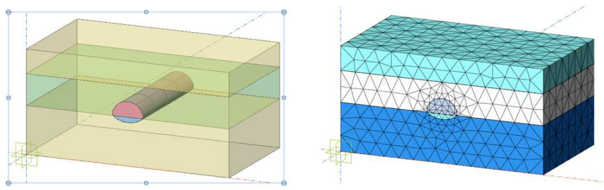

Figure 7. 3D model of the Petřiny station in Midas GTS

Table 5.Results for Petřiny station - settlement in mm.

Calc.	Final surface	Final station	Left side wall surface	Left side wall tunnel
1	6.5	10.2	2.2	4.1
2	4.9	11.1	2.5	4.7
3	5.9	13.0	3.4	6.2
4	6.9	13.0	1.7	3.4
5	6.6	13.8	2.4	6.6
average	6.2	12.2	2.4	5.0

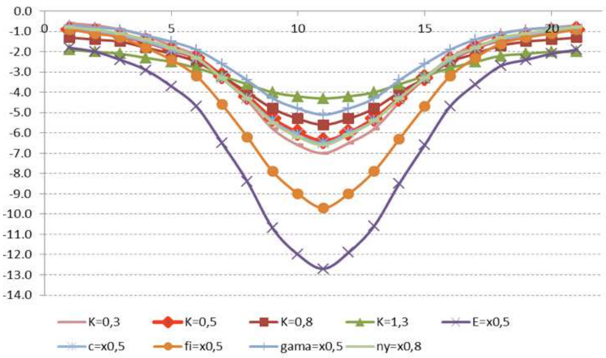

Figure 8. Vertical deformation of the surface perpendicular to the tunnel axis [mm], Red line – initial parameters, max. E * ½, min- settlement K_0=1.3

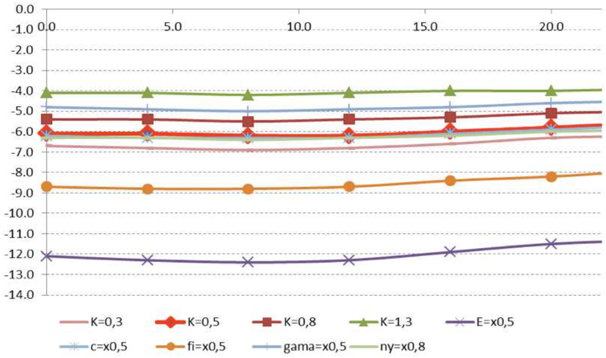

Figure 9. Vertical deformation of the surface along the tunnel axis [mm], Red line – initial parameters, max. E * ½, min- settlement K_0=1.3

5 Conclusion

The results of performed analyses show the effect of the variation of input parameters describing rock mass behaviour on the results of numerical modelling of various tunnels. The presented results demonstrate that the differences can be significant. The studies presented in the paper assume linearly independent input geotechnical parameters, therefore correlations of parameters were not considered. In addition, all of the above mentioned works consider the LHS-Median method type (the mid-point of an interval within the domain of the distribution functions). Further, normal probability distribution of input parameters was assumed. However, the importance for determination of the final structural behaviour of at least a basic study of the variation of the input parameters can be clearly seen.

The Latin Hypercube Sampling method is a procedure with advantages for the qualified statistical evaluation of FE calculations. This method makes significant time saving in comparison with other statistical methods (Monte Carlo method, estimations of probability moments, etc.).

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